Testing the Trade-off Theory of Capital Structure: A Kalman Filter Approach

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Abstract

In this paper, we use a Kalman filter in order to test the standard dynamic trade-off model of capital structure. In this model, the observed realized debt-equity ratio is a weighted average of the unobservable target debt-equity ratio and last period's realized debt-equity ratio. The use of the Kalman filter, however, allows us to directly estimate the unobservable target debt-equity ratio. We find that the trade-off model cannot be rejected for 32% to 52% of the firms in our sample at the standard 5% level. We also use a regression in order to test if our Kalman filter estimated target debt-equity ratios are related to the fundamental variables usually proposed in the corporate structure literature. Overall, we find support for our estimates.

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1. Introduction

The hypothesis that target debt-equity ratio are employed by corporations has been tested extensively in the corporate structure literature. Graham and Harvey (2001) find that 81% of firms use a specific (or range of) target debt-equity ratio(s) when making their debt decisions. Furthermore, Flannery and Rangan (2006) point out that most empirical analysis of this hypothesis rely heavily on the trade-off theory, which states that firms select a target debt-equity ratio by trading off their cost and benefits of leverage. The working version of the trade-off theory allows for the adjustment of the debt-equity ratio over time, rendering a dynamic trade-off model. Hovakimian, Opler, and Titman (2001), Strebulaev (2004), Flannery and Rangan (2006), and Kayhan and Titman (2007) find that the dynamic trade-off model dominates alternative models, such as: Myers' (1984) pecking order model, Baker and Wurgler's (2002) market timing model, and Welch's (2004) managerial inertia model. They conclude that firms actively pursue target debt-equity ratios over time even though market frictions lead to an incomplete adjustment in any one period. Fama and French (2002), however, do not find a clear cut dominant model.

The trade-off model literature recognizes that the target debt-equity ratio is empirically unobservable and, therefore, uses a reduced form equation to directly estimate the partial adjustment parameter, which is called the "speed of adjustment." Techniques such as two-stage estimation, instrumental variables, and dynamic panels are used in order to work around the fact that the debt-equity ratio is unobservable and get an estimate of the speed of adjustment. Yet, as reported by Flannery and Hankins (2007), the estimates obtained employing these methods exhibit great variation. For example, Fama

and French (2002) report annual estimates of the partial adjustment parameter from 7 to 18%, Roberts (2002) reports annual estimates close to 100% for some industries. These wide differences are attributed to econometric problems, among them, unobservable variable issues, heterogeneous panel, autocorrelated and cross correlated errors, short panels, unbalanced panels, etc.

In this paper, we estimate the structural dynamic trade-off model by employing the Kalman filter estimation technique. The main advantage of using the Kalman filter is that it allows us to estimate the unobserved target debt-equity ratio directly, thus, leading to a simple test of the trade-off capital structure theory. With these estimates, we test whether the firm's realized debt-equity ratio is equal to a weighted average of the target debt-equity ratio and last period's realized debt-equity ratio. Moreover, since there is no consensus regarding the dynamic behavior of the target debt-equity ratio, the use of the Kalman filter technique allows us to estimate the dynamic trade-off model under different assumptions regarding the dynamics of the unobservable debt-equity ratio. In our analysis we use an autoregressive process, a random walk process, and a constant process and show their impact on the results.

We further depart from the extant literature by not using panel data estimation, as it is often done in the recent literature. Instead, we estimate and test the structural dynamic models for individual firms. This focus on individual firms allows us to study the percentage of firms for which the dynamic trade-off model holds empirically, as well as to estimate the speed of adjustment for each firm.

Our paper is closely related to Roberts (2002), who also uses a Kalman filter model to estimate a dynamic trade-off model. He uses the Kalman filter to indirectly

estimate the target debt-equity ratio through a set of economic variables, while we use the Kalman filter to directly estimate the target debt-equity ratio. A significant difference between our approach and Robert's (2002) is that he emphasizes the speed of adjustment and its determinants, while we emphasize testing the trade-off model.

Our empirical analysis indicates that the dynamic trade-off model holds –i.e., cannot be rejected at the standard 5% level- for 32% to 52% of the firms in our sample, depending on the assumptions about the target debt-equity process used to estimate the Kalman filter. We also find that for the model assuming an autoregressive target debt-equity ratio, the median and the average quarterly speed of adjustment are .161 and .276, respectively. These numbers are close to the annual estimates reported in Flannery and Rangan (2006). Confirming previous work, we find a huge cross-sectional variation in the speed of adjustment parameter. The empirical 95% confidence interval for the speed of adjustment has as bounds .025 and .951. The interquartile range, however, is not that extreme, going from .088 to .347.

The rest of the paper is organized as follows. Section 2 presents the model and the methodology of our test of the dynamic trade-off model. Section 3 presents the data, Section 4 presents the results and Section 5 concludes.

2. The Model

The dynamic trade-off model is based on the idea that firms cannot instantaneously achieve their target leverage, rather they adjust their realized debt-equity ratios over time. Thus, every time period the firm uses the last period's difference between the realized debt-equity ratio and its target debt-equity ratio in oder to achieve a

more desirable debt-equity ratio in the next period. The dynamic trade-off theory is described by the following model:

$$\Delta D_{i,t} = \gamma_i \left(D^*_{i,t} - D_{i,t-1} \right) + e_{i,t} \tag{1}$$

where $D_{i,t}$ is firm i's realized debt-equity ratio in period t, $D^*_{i,t}$ is firm i's target debt-equity ratio, Δ is the difference operator, γ_i is the partial adjustment coefficient; $0 \leq \gamma_i \leq 1$, and $e_{i,t}$ is a regression error.

Since the target debt-equity ratio is unobservable, it is not possible to directly test the dynamic trade-off model in equation (1) and it is common to model the target debt-equity ratio, $D^*_{i,t}$, as a linear function of a set of economic variables. The following equation completes the standard empirical setup for the trade-off model:

$$D^*_{i,t} = \beta' X_{i,t} \ , \tag{2}$$

where the vector $X_{i,t}$ contains a set of widely studied variables in the literature such as earnings before taxes, market-to-book ratio, marginal tax rate, Altman Z score, industry dummy variables, capital expenditure, research and development expenditures, etc. We emphasize that equation (2) is not part of the trade-off theory, since the trade-off theory does not explicitly model the target debt-equity ratio. Rather equation (2) is an ad-hoc formulation where some explanatory variables, which are derived from different theories and other explanatory variables, included because they fit the data. (See, for example, Rajan and Zingales (1995), Fama and French (2002), Chen and Zhao (2005).)

Substituting (2) into (1) yields:

$$D_{i,t} = \gamma_i \beta' X_{,i,t} + (1 - \gamma_i) D_{i,t-1} + e_{it}, \tag{3}$$

which is the standard framework used in the literature to estimate capital structure models.

Notice that the test of the trade-off theory would be straightforward if an estimate of the target debt-equity ratio were available. Simply, rearrange equation (1) to obtain:

$$D_{i,t} = \gamma_i D_{it}^* + (1 - \gamma_i)D_{i,t-1} + e_{it} \tag{4}$$

Equation (4) tells us that if the standard partial adjustment version of the trade-off model is correct, then the realized debt-equity ratio is a weighted average of its lagged debt-equity ratio and the target debt-equity ratio. If $D_{i,t}^*$ is available, then, to test the trade-off model in equation (4) we only need to test that the slope coefficients in a linear regression of $D_{i,t}$ against $D_{i,t}^*$ and $D_{i,t-1}$ add up to 1. Unfortunately, the usual estimation of equation (3) does not allow the researcher to test this hypothesis.

As mentioned above, given that the target debt-equity ratio is unobservable, many papers study the speed of adjustment parameter, γ_i , assuming a common γ for all the firms ($\gamma_i = \gamma$ for all i) by employing a panel regression of realized debt-equity ratio on its one-period lag as well as a vector of variables $X_{i,t}$; see, for a recent example, Flannery and Hankins (2007).

The estimation of equation (3), however, raises two main problems: the identification problem, and the firm heterogeneity of the sample problem. The identification problem arises because equation (3) is a reduced form equation that depends on the correct specification of equation (2). It follows that while equation (3) can be used to estimate the partial adjustment coefficient, γ, it cannot be used to estimate $D_{i,t}^*$ directly nor can it be used to test the trade-off model.[1] In other words, even when the coefficients in equation (3) are statistically significant, one may only infer that a linear

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[1] Several papers conflict regarding the interpretation of the results from the estimation of equation (3). In particular, a significant speed of adjustment coefficient can be obtained under different theories. See, for example, Chen and Zhao (2005).

regression of the realized debt-equity ratio on the lagged (observed) debt-equity ratio and the driving variables $X_{i,t}$ produces significant results. One cannot draw any conclusion regarding the validity of equations (1) and/or (2). Note that the unobservable D_{it}^* may be estimated in a second step through the indirect estimation of β in equation (3). But, we should keep in mind that a correct specification of equation (2) is crucial to draw valid inferences about the trade-off model. Therefore, in trying to avoid this possible misspecification issue, different studies assume the $\gamma_i=\gamma$ for all i, estimate γ'β and focus attention on γ. That is, they do not estimate β or D_{it}^*, and hence do not directly test the dynamic trade-off model.

The firm heterogeneity of the sample problem arises since panel methods are used to estimate equation (3). Panel methods assume a common γ for all firms (see, for example, the use of Fama-Macbeth's method in Fama and French (2002) or the use of fixed effects in Flannery and Rangan (2006).) However, the significant cross-sectional variation of debt-to-equity ratios reported in the literature clearly indicates that assuming a common partial adjustment coefficient for all firms is a extremely restrictive assumption.

In this paper we overcome both problems by employing the Kalman filter technique and, thus, estimating the unobservable target debt-equity ratio directly. As will become clear below, the target debt-equity ratio, $D_{i,t}^*$, can be directly estimated using a Kalman filter. First, we assume that $D_{i,t}^*$ follows an AR(1) process. This assumption leads to the following state-space model:

$$D_{i,t} = e_{i,t} + [\gamma_i \quad 1-\gamma_i]\begin{bmatrix} D_{i,t}^* \\ D_{i,t-1} \end{bmatrix} \qquad (5A)$$

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$$\begin{bmatrix} D_{i,t}^* \\ D_{i,t-1} \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ \gamma_i & 1-\gamma_i \end{bmatrix} \begin{bmatrix} D_{i,t-1}^* \\ D_{i,t-2} \end{bmatrix} + \begin{bmatrix} u_{i,1t} \\ u_{i,2t} \end{bmatrix} \quad , \tag{5B}$$

where $e_{i,t}$, $u_{i,1t}$, and $u_{i,2t}$ are independent normally distributed error terms. In the state-space model terminology, equation (5A) is called the *measurement equation*, while equation (5B) is called the *state equation*. The basic tool used to estimate state-space models is a Kalman filter, which is a recursive procedure that estimates the unobserved component or the state vector. (See Hamilton (1994).) Roberts (2002) also uses a Kalman filter to estimate the dynamic trade-off model, assuming that the variables in equations (1) and (2) are latent. In our approach, the only latent variable is $D_{i,t}^*$, which allows us to directly test the dynamic trade-off model.

We use the following unrestricted form of model (5A)-(5B):

$$D_{i,t} = e_{i,t} + \begin{bmatrix} \gamma_{i1} & \gamma_{i2} \end{bmatrix} \begin{bmatrix} D_{i,t}^* \\ D_{i,t-1} \end{bmatrix} \tag{6A}$$

$$\begin{bmatrix} D_{i,t}^* \\ D_{i,t-1} \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ \gamma_{i1} & \gamma_{i2} \end{bmatrix} \begin{bmatrix} D_{i,t-1}^* \\ D_{i,t-2} \end{bmatrix} + \begin{bmatrix} u_{i,1t} \\ u_{i,2t} \end{bmatrix} \tag{6B}$$

We emphasize that the only input needed to estimate the structural dynamic trade-off model (6A)-(6B) is the realized debt-equity ratio, $D_{i,t}$, and that model (6A)-(6B) affords us the simultaneous estimation of $D_{i,t}^*$ and the parameters ϕ, γ_{i1} and γ_{i2}, along with the covariance matrix for the error terms. Based on these estimates, we test the dynamic trade-off model directly by testing that γ_{i1} and γ_{i2} in equation (4) add up to 1. Moreover, if the dynamic trade-off model is not rejected, we can use equations (6A) and (6B) to

estimate the target debt-equity ratio over time for each firm, along with the firm's speed of adjustment parameter, γ_{i1}.

This approach avoids the problems associated with endogeneity, which is a common problem in the empirical models of capital structure. For example, many of the economic variables that determine the target debt-equity in equation (2) are simultaneously determined with the firm's leverage. As pointed out by Roberts (2002), ignoring the endogeneity issue leads to a well-known, but seldom-addressed, biasing of coefficients in the standard regression framework.

Finally, notice that model the dynamic trade-off model, described in Equation (2), allows for the target leverage ratio to change over time. This formulation is consistent with capital structure theory that posits that the target leverage for a firm changes over time as the characteristics of the firm change. (See, for example, Hennessy and Whited (2005) and Titman and Tsyplakov (2005).) Other researchers, however, assume that the target levarage ratio is constant. (See, Collin-Dufresne and Goldstein (2001).) In spite of the different assumptions, it is commonly found that observed leverage ratios show mean reversion. Moreover, while Marsh (1982), Auerbach (1985) and Opler and Titman (1995), among others, document that companies tend to gradually adjust their capital structures toward a target level of leverage; Jalilvand and Harris (1984) find that leverage ratios are reasonably stable over time. More recently, Drobetz, Pensa, and Wanzenried (2007) find that book leverage over the time period of 1983-2005 was quite stable around .6, though market leverage tended to be more time-varying. Roberts (2002) presents estimates employing a constant and a time-varying process for the target leverage and finds that the parameter estimates are similar in both cases.

In light of the mixed evidence, we test the dynamic trade-off model under several assumptions. First, we assume that the target debt-equity ratio follows an AR(1) process. Second, we assume that the target debt-equity ratio is constant. Finally, as a robustness check, we also assume a third scenario, under which the target debt-equity ratio follows a random walk process, making the target debt-equity ratio completely unpredictable based on previous information.

3. The Data

Several definitions of the debt-equity ratio are used in the literature. In our analysis, we use the following definitions: Debt is the book value of the firm's long term debt and Equity is the market value of a firm's common stock.   We   use   long-term debt since the trade-off theory argues that the partial adjustment is due to the existence of transaction costs or other market imperfections. Short-term debt tends to be more flexible than long-term debt, therefore, a partial adjustment mechanism is not that theoretically appealing. Moreover, since we use quarterly data, a lot of short-term dynamics may be lost between quarters.[2]

The use of book value debt vs. market value debt is also a common issue in the literature. Marsh (1982) presents an early discussion of this issue, finding that his empirical results are not significantly affected by the measurement choice. More recently, Drobetz, Pensa, and Wanzenried (2007) present an updated summary of the pros and cons of both measures. According to their discussion, using market values may not reflect

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[2] Flannery and Rangan (2006) use three definitions of debt, including total liabilities, long-term debt plus short-term debt, and long-term debt only. They find their results to be similar across the different debt definitions.

the underlying changes initiated by the firm's decision makers. They add that from a more pragmatic point of view, the market value of debt is often not readily available and the calculation of market values of debt is cumbersome. They end-up referring to the market value of debt as "quasi-market" value and they run their empirical analysis with book values and quasi-market values of debt. They conclude that firms are more concerned with book leverage ratios than with market leverage ratios.

The empirical literature estimates equation (2) using the following variables: Volatility of cash flows, Product uniqueness, Tangible assets, Size, Profitability, Capital expenditures, Market-to-book ratio, Z score, Capital expenditure, Cash position, Tax shield, Tax rates, and Mitigation of free cash flow problem. In the Appendix, we present the exact definitions of these variables, along with their respective COMPUSTAT items. We use these variables to check the quality of the Kalman filter estimates of the target debt-equity ratio.

Our sample consists of quarterly data for the period of 1985:I to 2005:IV. The data is obtained from COMPUSTAT. All the firms in our sample have ininterrupted observations in the sample period.[3] Following the standard practice in the literature, we exclude financials and regulated industries. Our sample size is 578 firms.

Table I presents the univariate statistics for the debt-equity ratio for the firms in our sample. The average and median debt-equity ratios are .279 and .260. For close to 40% of the firms there is evidence of significant skewness, while for 20% of the firms

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[3] We only use firms with continuous observations to avoid the problems associated with missing data. Roberts (2002) finds that missing data impact the magnitude and statistical significance of the estimates, but not the direction of association between variables. Since the magnitude and statistical significance of the tests is crucial for our test, we avoid firms with missing data.

there is evidence of significant excess kurtosis. For all firms, the debt-equity ratio is highly autocorrelated, with an average autocorrelation coefficient of .89. The null hypothesis of no autocorrelation of order 4 is rejected for all firms by the LB(4) statistics for all standard significance levels.

Table II presents some descriptive statistics for the variables that are often used in the literature to explain the behavior of the debt-equity ratio.

4. Empirical Analysis

As mentioned above, most empirical works estimate of equation (3) using a panel data technique which yields the average speed of adjustment. From a statistical point of view the panel setting is relatively powerful. It ignores, however, the heterogeneity in the individual firms' parameters. Our choice of estimation method allows us to test the trade-off theory for each firm, instead of testing the trade-off theory only for the average firm. First, we conduct an unrestricted estimation of (6A) and (6B). Second, we conduct a restricted estimation by imposing the restriction that γ_1 and γ_2 sum up to 1. Then, we construct a likelihood ratio test statistics in order to test the dynamic trade-off model. For this test, the null hypothesis is H$_0$: $\gamma_1 + \gamma_2 = 1$.

Notice, however, that the trade-off model in equation (6A) makes no sense when γ_1 is equal to zero. Therefore, we use a two-step process to decide whether the dynamic trade-off model is appropriate. First, we use a t-test to test the null hypothesis that $\gamma_1 = 0$. If this null hypothesis cannot be rejected, the trade-off model can be rejected directly,

without testing the null hypothesis implied in equation (6A). Second, we do the above mentioned likelihood ratio test to test the null hypothesis of γ_1 and γ_2 summing up to 1.[4]

Table III presents the results for the unrestricted estimation. In panel A, we find that the trade-off model holds –i.e., cannot be rejected at the standard 5% level- for 32% of the firms in our sample. This proportion increases to 52% for the constant scenario for $D_{i,t}^{*}$.

We also estimate the speed of adjustment for each firm in our sample.[5] For the case in which $D_{i,t}^{*}$ follows an AR(1) process, the median and the average quarterly speed of adjustment, γ_1, are .161 and .276, respectively. These quarterly numbers, once compounded, are similar to the panel data annual estimates reported by Jalivand and Harris (1984), Alti (2006) and Flannery and Rangan (2006), which are in the .30 to .56 range; but, the estimates are low relative to the quarterly industry estimates reported by Roberts (2002). Confirming Roberts (2002), however, we find a big variation in the estimated speed of adjustment with a 90% confidence interval whose bounds are .951 and .0255. The interquartile quarterly range is (.347 - .088), again, once compounded, not that far from the annual estimates reported in the literature. It is also possible to estimate γ_1 through γ_2, by imposing $\gamma_1+\gamma_2=1$, as typically done in the literature when equation (3)

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[4] Given our small sample size, 84 observations, we expect to have relatively large standard errors, which will lead to a larger number of non-rejections of H_0, using the standard asymptotic t-values. We decided to use the 10% level –i.e., 1.645- for the t-tests in the first-step- instead of the more standard 5% level to test H_0. In a small simulation, we found that at the 10% level, the t-test correctly rejected 92% of the time H_0: γ_1 =0.

[5] Note that we can directly estimate the speed of adjustment, γ_1, while the literature estimates an unrestricted γ_2 and then imposes the restriction $\gamma_1=(1-\gamma_2)$ to get an estimate of γ_1.

is estimated. In this case, the median and average quarterly speed of adjustment are .168 and .307, respectively.

For the case in which $D_{i,t}^*$ is constant the median and the average quarterly speed of adjustment are .117 and .094, respectively. Again, we find significant variation in the estimated speed of adjustment with a 90% confidence interval whose bounds are .413 and .026. The interquartile quarterly range is (.182 - .069). Again, calculating γ_1 through γ_2, we get a median and an average quarterly speed of adjustment equal to .110 and .129, respectively.

Observe that the estimates obtained under the constant scenario for $D_{i,t}^*$ are more stable than those obtained under the AR(1) scenario. It is, thus, important to test for how many firms the AR(1) case applies. This can be done with the Wald test, with the null hypothesis H_0: $\phi = 0$. We find that this hypothesis is rejected for 62% of the firms at the 5% level. This result is not that surprising given the empirical distribution reported in Table III, Panel A.

In Panel B, Table III, we present the estimates of model (6A)-(6B), only for the firms for which the trade-off theory cannot be rejected. Again, for the AR(1) case scenario for $D_{i,t}^*$, we find a good dispersion of estimates; but, in general, with higher estimates for the speed of adjustment, γ_1, and, as expected, lower estimates for γ_2.[6] For the constant case scenario for $D_{i,t}^*$, we find more stable estimates, which are a little bit lower, but overall similar to the ones reported on Panel A. For more than half the sample we find evidence supporting the constant case scenario for D_{it}^*.

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[6] The interquartile range for γ_2 is given by (.878-.215). It is a little bit misleading, since the .95-.32 range is (.878-.611).

In Panel C, Table III, we present the estimates of model (6A)-(6B), only for the firms for which the trade-off theory is rejected. For the AR(1) case, the estimates tend to be more stable, showing an interquartile range for γ_1 equal to (.258, .060), closer to the annual estimates reported by the empirical literature. Again, the estimates for the constant case scenario for $D_{i,t}^{*}$ are more stable.

Based on the results for the dynamic trade-off model for $D_{i,t}^{*}$ presented in Table III, we conclude that while some significant proportion of firms use a target debt-equity ratio, the majority of the firms do not. This result may be attributed to our use of the Kalman filter technique. One way to check our estimates of $D_{i,t}^{*}$ is to see if they are correlated with the standard set of economic variables that are proposed by the trade-off theory –i.e., the set of variables used in equation (2). If the Kalman filter produced reasonable estimates, then these estimates should be highly correlated to the set of fundamental variables only for the group of firms for which the dynamic trade-off model cannot be rejected. Therefore, we run a cross-section regression version of equation (2) using our Kalman filter estimated debt-equity ratios as the dependent variable and as the explanatory variables a set of financial variables. That is, we estimate:

$$\hat{D}_i^{*} = \alpha + \delta' X_i + \xi_i, \qquad\qquad (7)$$

where $\hat{D}_i^{*}$ is the Kalman filter estimated debt-equity ratio, X_i is the vector of explanatory variables and ξ_i is the error term. We estimate equation (7) using as explanatory variables Volatility of cash flows, Tangible assets, Firm size (sales divided by total sales of sample), Profitability (net operating income), Altman's Z score, Capital expenditures, Market-to-book ratio, Cash and short-term marketable securities, Tax shields

(depreciation and amortization), Income tax rate, and Mitigation of free cash problem (after tax operating income).[7] The majority of the variables are scaled by total assets –see Appendix for exact definitions.

Before estimating (7), we divide the firms into two groups. Group A includes the firms for which the dynamic trade-off model cannot be rejected and Group B consists of all the firms for which the dynamic trade-off model is rejected. As explained above, if we have correctly estimated the trade-off model, then the target debt-equity ratio estimates for Group A should be explained by the proposed set of economic variables, while the target debt-equity ratio estimates for Group B should be largely uncorrelated with any variables.[8]

Table IV presents the results of both regressions. It is likely that these regressions suffer from multicollinearity, but since multicollinearity affects the standard errors of the coefficients and not the coefficients themselves, we focus on the overall explanatory power given by the R^2. For Group A, the set of explanatory variables explains 55% of the variability of the Kalman filter estimated target leverage. For Group B, however, the same variables only explain 8.2% of the variability of the Kalman filter estimated target leverage. The evidence in Table IV is quite strong in favor of the Kalman filter estimates of $D_{i,t}^*$. We find that the Kalman filter estimated target debt-equity ratios show a high correlation with the set of fundamental variables only for the group of firms for which the dynamic trade-off model cannot be rejected (Group A).

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[7] See Fama and French (2002), Roberts (2002), Flannery and Rangan (2006), Drobetz, Pensa, and Wanzenried (2007).
[8] A non-zero correlation may exist. For example, if the estimated target debt-equity is correlated with observed debt-equity ratio.

Even though we emphasize the joint explanatory power, a look at Table IV sheds some light on the relation between profitability and the debt-equity ratio. Fama and French (2002) point out that the negative relation between profitability and the debt-equity ratio is significant scar for the trade-off theory. Table IV, however, shows that the negative relation between profitability and the debt-equity ratio is only significant for the firm where the trade-off theory is rejected. For the firms that the trade-off theory is not rejected there is a positive, though not statistically significant relation between profitability and the debt-equity ratio.

5. Conclusions

In this paper, we present a test of the dynamic trade-off model of capital structure. We depart from the standard estimation technique of the trade-off model, by directly estimating a structural model, instead of the standard reduced form equation. Given that in the structural trade-off model, the target debt-equity ratio is unobservable, the state-space representation is a natural model to test the dynamic trade-off model. We use a Kalman filter, which allows us to directly estimate the unobserved firm's target debt-equity ratio.

Under the structural model of the trade-off theory, the firm's observed, or realized, debt-equity ratio is a weighted average of last period's realized debt-equity ratio and the firm's target debt-equity ratio. With the estimated model parameters and the estimated target debt-equity ratio, we suggest a simple test of the trade-off model. This test checks if the estimated parameters in the structural model foe each firm add up to one.

The focus on individual firms allows us to directly study the number of firms in which the dynamic trade-off model cannot be rejected. In our sample of 578 firms, we find that the trade-off model holds –i.e., cannot be rejected at the standard 5% level- for 32% when we assume that the target debt-equity ratio follows an AR(1) model. The model holds for as many as 52% when the target debt-equity ratio is assumed to be constant. We also estimate the speed of adjustment for each firm. The median and the average speed of adjustment are .161 and .276, respectively. These numbers are close to the annual estimates reported in Flannery and Rangan (2006). Confirming Roberts (2002), we find a huge cross-sectional variation in the speed of adjustment parameter. The empirical 95% confidence interval for the speed of adjustment has as bounds .025 and .951. The interquartile range, however, is not that extreme, going from .088 to .347.

References

Alti, A., 2006, How persistent is the impact of market timing on capital structure?, *Journal of Finance*, 61, 1681-1710.

Baker, M., and J. Wurgler, 2002, Market timing and capital structure, *Journal of Finance,* 57, 1-32.

Chen, L., and X. Zhao, 2005, Profitability, mean reversion of leverage structure choices, *Working paper*, Michigan State University.

Collin-Dufresne, P., and R. Goldstein, 2001, Do credit spreads reflect stationary leverage ratios? *Journal of Finance*, 56, 1929–1957.

Drobetz, W., P. Pensa, and G. Wanzenried, 2007, Firm characteristics, economic conditions and capital structure adjustments, W*orking Paper,.* University of Hamburg.

Fama, E., and K. French, 2002, Testing the trade-off and pecking order predictions about dividends and debt, *Review of Financial Studies*, 15, 1-33.

Fama, E., and J. D. MacBeth, 1973, Risk, return, and equilibrium: empirical tests, *Journal of Political Economy*, 81, 607-636.

Faulkender, M., M. Flannery, K. Hankins and J. Smith, 2007, Are Adjustment costs impeding realization of target capital structure?, *Working Paper.*

Flannery, M. and K. Hankins, 2007, A theory of capital structure adjustment speed, *Working paper.*

Flannery, M., and K. Rangan, 2006, Partial adjustment toward target capital structures, *Journal of Financial Economics,* 79, 469-506.

Graham, J. R., and C. R. Harvey, 2001, The theory and practice of corporate finance: Evidence from the field, *Journal of Financial Economics*, 60, 187-243.

Hamilton, J. D. (1994), Time Series Analysis, Princeton, NJ: Princeton University Press.

Hennessy, Christopher A, and T. M. Whited, 2005, Debt Dynamics, *Journal of Finance*, 60, 1129–1165.

Hovakimian, A., T. Opler, and S. Titman, 2001, The debt-equity choice, *Journal of Financial and Quantitative Analysis*, 36, 1-24.

Jalilvand, A. and R. Harris, 1984, Corporate behavior in adjusting to capital structure and dividend targets: An econometric study, *Journal of Finance*, 39, 127-145.

Kayhan, A. and S. Titman (2007). Firms' histories and their capital structures. *Journal of Financial Economics,* 83, 1-32.

Lemmon, M., M. Roberts, and J. Zender, 2007, Back to the beginning: Persistence in the cross-section of corporate capital structure, *Journal of Finance*, forthcoming.

Marsh, P., 1982, The Choice between equity and debt: an empirical study, *Journal of Finance*, 37, 121-44.

Myers, S. C., 1977, Determinants of Corporate Borrowing, *Journal of Financial Economics*, 5, 147–175.

Myers, S. C.. 1984, "The Capital Structure Puzzle," *Journal of Finance*, 39, 575-592

Rajan, R., and L. Zingales, 1995, What Do We Know about Capital Structure? Some Evidence from International Data, *Journal of Finance,* 50, 1421-1460.

Roberts, M., 2002, The dynamics of capital structure: An empirical analysis of a partially observable system, *Working Paper, Duke University*.

Titman, S. and S. Tsyplakov, 2005, A dynamic model of optimal capital structure, *Working paper*, Univeristy of Texas at Austin.

Strebulaev, I., 2004, "Do tests of capital structure theory mean what they say?" *Working Paper*, London Business School.

Strebulaev, I., and B. Yang, 2006, The mystery of zero-leverage firms, *Working paper*, Stanford Graduate School of Business.

Welch, I., 2004, Capital structure and stock return, *Journal of Political Economy*, 112, 106–131.

APPENDIX: Variable Definitions

1. Bankruptcy Costs and Cost of Financial Distress Variables

i.   Volatility of Cash Flows: Absolute Value of Change in Net Income / Total Asset

COMPUSTAT definition: |Data69(t) – Data69(t-1)| / Data 44

ii.   Product Uniqueness: Selling Expenses / Total Sales

COMPUSTAT definition: Data1 / (Data1+Data2)

iii.   Tangible Assets: (PP&E) / Total Assets

COMPUSTAT definition: Data42 / Data44

iv.   Firm Size: Sales / Total Sales of Sample

COMPUSTAT definition: (Data1+Data2) / Cross Sectional Sum of (Data1+ Data2)

iv.   Profitability: Net Operating Income / Total Assets

COMPUSTAT definition: (Data21-Data5)/ Data44

v.   Z-Score: 3.3 EBIT / TA + Sales / TA + 1.4 Retained Earnings / TA + 1.2 Working Capital/ TA

COMPUSTAT definition: 3.3x(Data21-Data5+Data31) / Data44 + (Data1+Data2) / Data44+1.4Data58 / Data44 + 1.2(Data40-Data49)/Data44

vi.   Capital Expenditure: Capital Expenditures (t+1) / Total Assets (t+1)

COMPUSTAT definition: Data90(t+1) / Data44(t+1)

vii.   Market to Book: (Total Assets – Book Equity + Market Equity) / Total Assets

COMPUSTAT definition: (Data44 – Data59 + Data61*Data14) / Data44

COMPUSTAT definition: (Data61*Data14) / (Data44 – Data49 – Data51)

viii.   Cash & Short Term Marketable Securities

COMPUSTAT definition: Data36

2. Tax Variables

i.   **Tax Shield:** (Depreciation and Amortization) / TA

COMPUSTAT definition: Data5 / Data44

ii.   **Tax Rate:** Income Tax Rate

COMPUSTAT definition: Data 6 / (Data6 + Data69)

3. Mitigation of Free Cash Flow Problem

i. Mitigation: After Tax Operating Income / Total Assets

COMPUSTAT definition: (Data21 – Data5) / Data44

TABLE I
Univariate Statistics for Debt-Equity Ratio

	Max	3rd Quant	Median	1st Quant	Min		Mean	St Dev
Mean	0.836	0.391	.0260	0.149	0.005		0.279	0.163
St. Dev	0.290	0.150	0.097	0.063	0.004		0.110	0.058
Skew	3.788	0.841	0.460	0.049	-1.809		0.463	0.678
Ex Kurt	20.460	0.265	-0.453	-0.880	-1.713		-0.217	1.681
Rho	0.976	0.922	0.888	0.832	0.533		0.868	0.077
LB(4)	308.42	241.70	200.41	154.70	45.431		195.87	57.333
LBS(4)	312.41	225.37	182.76	137.01	11.742		179.22	59.556

Notes:
St. Dev: Standard Deviation.
Skew: Skewness.
Ex Kurt: Excess Kurtosis.
Rho: First-order autoregressive coefficient.
LB: Ljung-Box (1978) statistic for levels. It follows a $\chi^2(4)$.
LBS: Ljung-Box (1978) statistic for squared series. It follows a $\chi^2(4)$.

TABLE II
Statistics for Firm Characteristic Variables

	VCF	Punique	TangibA	FSize	Profit	Zcore
Mean	0.0125	0.0494	0.3928	0.002	0.0217	0.825
Std Dev	0.0341	0.0796	0.2579	0.005	0.0240	0.756
Median	0.0046	0.0303	0.3482	0.001	0.0210	0.835
Minimum	0.0000	-0.0340	0.0000	0.000	-1.0418	-19.15
Maximum	1.6390	2.4279	1.9523	0.075	0.4786	5.892
Interquartile	0.0097	0.0097	0.4163	0.0017	0.0207	0.834
95th p	0.0452	0.1644	0.8295	0.0078	0.0540	1.804
5th p	0.0002	0.0000	0.0130	0.000	-0.0046	0.057

	Capex	MB	Cash	TaxSh	TaxRate	Mitigation
Mean	0.0375	1.4879	0.0653	0.0112	0.3511	0.0156
Std Dev	0.0424	0.9514	0.0915	0.0072	0.1094	0.0204
Median	0.0248	1.2338	0.0316	0.0103	0.3618	0.0156
Minimum	-0.2164	0.3098	-0.0096	-0.0005	0.0002	-1.0418
Maximum	0.9024	54.3581	0.8822	0.3095	0.9892	0.4321
Interquartile	0.0398	0.5182	0.0693	0.0070	0.0906	0.0136
95th p	0.1147	2.8330	0.2446	0.0232	0.4945	0.0378
5th p	0.0001	0.8942	0.0022	0.0010	0.1522	-0.0033

Notes:
VCF: Volatility of Cash Flows.
Punique: Product Uniqueness.
TangibA: Tangible Assets.
FSize: Firm Size.
Profit: Profitability.
Zscore: Z-Score.
Capex: Capital Expenditure.
MB: Market to Book Value.
Cash: Cash & Short Term Marketable Securities.
TaxSh: Tax Shield.
Tax Rate: Income Tax Rate.
Mitigation: Mitigation of Free Cash Flow Problem

See Appendix for data definitions.

TABLE III
Distribution of Unrestricted Estimates of Model (6A)-(6B) for the different scenarios

Panel A. All Firms

	AR(1) Scenario						Constant Scenario				
	.95	.75	.50	.25	.05		.95	.75	.50	.25	.05
γ_1	.951	.347	.161	.088	.025		.413	.182	.117	.069	.026
SD(γ_1)	.186	.203	.210	.047	.064		.125	.082	.046	.044	.038
γ_2	.953	.908	.832	.659	-.015		.969	.928	.890	.834	.712
SD(γ_2)	.043	.046	.193	.108	.141		.031	.041	.054	.061	.076
ϕ	.987	.889	.268	-.066	-.921		-	-	-	-	-
SD(ϕ)	.059	.059	.195	.117	.041						
Proportion of Rejections of TM					68%		Proportion of Rejections of TM				48%
Proportion of Rejections of H$_0$: ϕ=0					62%						

Panel B. Trade-off Model Firms

	AR(1) Scenario						Constant Scenario				
	.95	.75	.50	.25	.05		.95	.75	.50	.25	.05
γ_1	1.174	.541	.256	.158	.110		.379	.222	.153	.111	.069
SD(γ_1)	.185	.170	.130	.074	.004		.087	.082	.062	.055	.049
γ_2	.921	.878	.811	.215	-.028		.938	.905	.863	.825	.720
SD(γ_2)	.044	.052	.079	.327	.227		.035	.046	.056	.060	.075
ϕ	.978	.885	.233	-.001	-.870		-	-	-	-	-
SD(ϕ)	.094	.106	.142	.117	.080						
Proportion of Rejections of H$_0$: ϕ=0					48%						

Panel C. Non Trade-off Model Firms

	AR(1) Scenario						Constant Scenario				
	.95	.75	.50	.25	.05		.95	.75	.50	.25	.05
γ_1	.693	.258	.124	.060	.020		.429	.115	.072	.043	.020
SD(γ_1)	.046	.165	.080	.062	.032		.0113	.073	.054	.054	.038
γ_2	.963	.921	.852	.700	.003		.974	.949	.923	.869	.691
SD(γ_2)	.085	.040	.049	.306	.134		.028	.034	.041	.043	.075
ϕ	.992	.891	.460	-.098	-.963		-	-	-	-	-
SD(ϕ)	.042	.059	.222	.134	.077						
Proportion of Rejections of H$_0$: ϕ=0					67%						

TABLE IV
Relation between Estimated Target Debt-Equity Ratio and Fundamental Variables

Variable	Trade-off Non Rejected			Trade-off Rejected	
	Coeff	t-stat		Coeff	t-stat
Intercept	42.17688	15.35		155.5174	14.88
Volatility of Cash Flows	40.97251	1.99		-57.3309	-0.67
Product Uniqueness	20.71077	1.65		-172.962	-4.47
Tangible Assets	36.99936	8.62		-76.9699	-1.4
Firm Size	-1.83906	-0.03		2733.874	0.83
Profitability	9.55753	0.14		-1297.53	-4.08
Z-Score	-15.6844	-13.01		-13.7299	-3.58
Capital Expend	-17.5049	-2.16		-131.476	-2.28
Market-to-Book ratio	-10.7152	-12.82		-7.86198	-4.28
Cash & ST Securities	6.16444	2.25		-35.246	-1.77
Tax Shield	379.2949	2.94		-151.047	-0.35
Tax Rate	-0.64618	-0.14		-12.3791	-0.64
Mitigation	72.29715	1.99		880.849	2.27
R^2	0.5523			0.0817	

This Table presents the results from the following pooled regression:

$$D_{i,t}^* = \alpha + \beta' X_{i,t} + \varepsilon_{i,t}$$

where D_{it}^* is the Kalman filter estimated target debt-equity ratio for firm i, assuming an AR(1) scenario, and $X_{i,t}$ is the set of fundamental variables for firm i described in the Appendix, and $\varepsilon_{i,t}$ is the error term. The regression is estimated for two groups: a group of firms (where the trade-off model cannot be rejected (Group A) and a group of firms where the trade-off model is rejected.
Notes: See Appendix for data definitions.